

# A NEW EMPIRICAL PREDICTION APPROACH FOR GROUNDBORNE VIBRATION IN BUILDINGS

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When predicting the impact of groundborne noise and vibration from railways, current prediction approaches typically range from simple empirical calculations to numerical models solved with supercomputer clusters. Modern multi-storey buildings are complex structures, and vibration engineers of such buildings generally have access to detailed building information. However, implementing the geometry and material properties in computer models in appropriate detail, running the calculations and interpreting the output is often too expensive a task. Whilst simpler empirical prediction methods exist, these do not take into account the majority of building or room parameters. A new empirical prediction approach is presented which has been developed through analysis of measured data and parametric studies on 3D finite element models. The new prediction approach assumes that the principal modal frequencies of the building and floors are known; these can be estimated with relatively simple analytical or finite element models, or in the case of existing buildings through measurements. Prediction of vertical vibration at mid-span or column positions is allowed for in one-third octave frequency bands between 4 – 200 Hz, relative to a basement vibration level. The results may then be used as part of re-radiated noise calculations. Building vibration results obtained using the new empirical prediction approach show reasonable agreement with 3D finite element models and measured data, but additional comparisons are to be conducted with measurement data to refine the model terms further.

Keywords: groundborne, noise, vibration, buildings, railway

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## 1. Introduction

When considering noise from railways affecting the occupants of nearby buildings, it is usually the airborne acoustic component that is dominant, where sound propagates through the air and to the occupant via façade elements such as windows and ventilators. However, where trains run in tunnels or the acoustic performance of the façade is high, the dominant acoustic path is via vibration propagating through the ground and building structure. In this instance, vibration is radiated as sound within rooms.

The groundborne noise experienced by an occupant is notoriously difficult to predict with great accuracy, as it is highly dependent on a number of structural features. Nevertheless, consultants are increasingly expected to be able to calculate such values, and provide informed input to design teams on this type of acoustic transmission [1].

In general, there are two main approaches to the prediction process: simple empirical ones and complex numerical ones. Simple empirical approaches tend to be based on guidance found in well-known sources such as [2,3]. However, due to the simplified nature of this kind of approach, account cannot be taken of even basic design alterations such as room size or wall structure type. These alterations may be accommodated in large-scale numerical models and even in some ‘simpler’ analytical models (e.g. [4]), but consultants will often not have the time or expertise required to utilise these approaches.

It is convenient to separate the prediction process into two stages; the first to predict vibration within a room, and the second to predict the resultant sound pressure level. This paper describes the development of empirical terms for predicting room vibration, based on a parametric study of finite element (FE) building models.

## 2. Building model

The approach taken in this research has been to prepare a 3D finite element representation of a 'generic' multi-storey reinforced concrete frame building. This has been used as a default case for a study of the influence of several different structural parameters.

### 2.1 Finite element model

The model geometry and mesh were created with the commercial software package COMSOL Multiphysics, which was also used for solutions in the frequency domain (4-200 Hz, 1301 frequency points, with approximately logarithmic spacing). The default building model includes six storeys plus a basement storey. Structural shafts are also included in the base configuration. Whilst the default model is symmetric about the mid-line x and y axes, symmetry has not been used in the formulation. Vibration levels have been evaluated at each floor level; at each mid-span position and near each column. The excitation of the building is with a uniformly distributed force over the basement floor slab, with a vector that is equal in each lateral direction, and the magnitude in the vertical direction being twice that in the lateral. All corresponding results are normalised to the vibration in the basement (determined from averaging over the appropriate basement locations). The influence of the ground has not been explicitly included in the model, although material damping properties were chosen to provide overall building damping in line with experimental data.

Walls and floor slabs are modelled as shell elements; columns are modelled as solid elements. The default model contains 224,250 degrees of freedom. The Young's modulus of elasticity includes a complex component to account for structural damping.

The view of an operational deflection shape at 10.2 Hz is given in Fig. 1 and shows the general layout of the model. It is interesting to note that, at this frequency, vibration is greatest at the upper storeys of the building due to the fundamental building mode, and at the centres of the floor slabs due to the influence of floor modes. The modal frequencies of individual floor slabs are influenced by the boundary conditions so not all bays resonate at the same frequency.

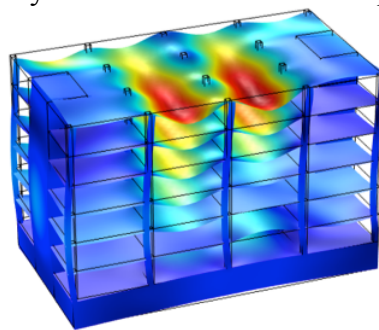


Figure 1 FE model of building, operational deflection at 10.2 Hz

The 3D model approach has been validated with measurements undertaken in existing buildings, with comparisons made at column and mid-span locations [5,6]. Whilst some differences were noted between measurements and modelled results, the general trends are represented well enough to support its use for further parametric study.

### 2.2 Parametric study

An extensive parametric study has been conducted which examines the influence of building geometry, material parameters and the presence of structural items such as shafts and internal walls. Only a limited set of results can be shown in this paper. The effect of the floor slab size is shown in

Fig. 2 for mid-span vertical vibration at the 3<sup>rd</sup> floor relative to vibration the basement level. The solid line with an asterisk represents the default model condition. Clearly, the floor slab dimensions strongly affect its modal response at low frequencies, but they can also influence the overall A-weighted vibration, as shown in Fig. 3. This is significant as it is the A-weighted vibration that would often be used to calculate and/or assess the resulting sound pressure level in rooms (e.g. [2,7,8]).

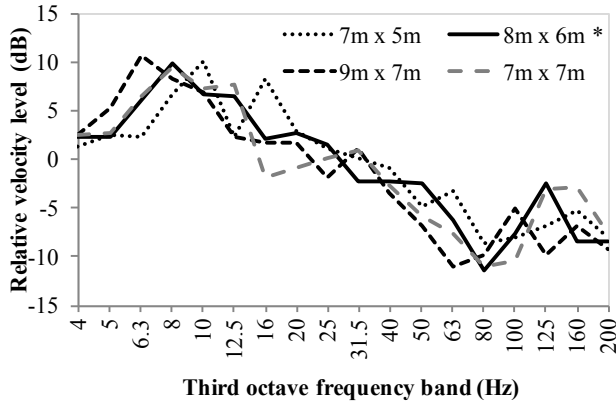


Figure 2 Mid-span vertical vibration at 3<sup>rd</sup> floor, relative to basement: different sizes of floor slab

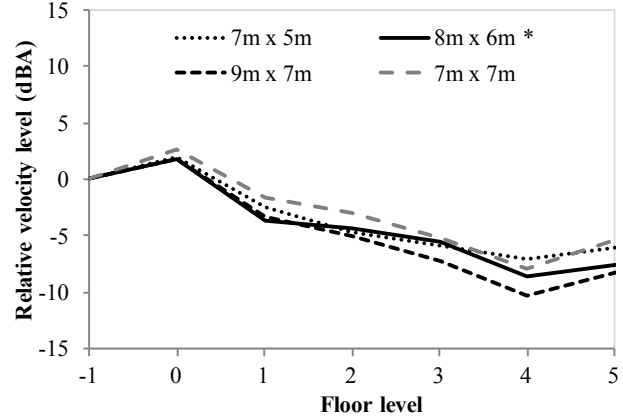


Figure 3 Mid-span A-weighted vertical velocity relative to basement: different sizes of floor slab

### 3. Derivation of empirical approach

From examining the results of the parametric FE model study, a number of trends were identified that have been used to develop empirical formulae. These formulae are intended for use with 1/3 octave ‘slow’ time-weighted data, or r.m.s. data with integration times exceeding about 0.5 seconds.

The vibration level at a point on a floor in the building is primarily associated with building modes and floor plate modes. It is therefore appropriate that these should be considered when developing new predictions.

The vibration level at column locations,  $L_{v,col}$  for a given 1/3 octave frequency band  $f$  (in Hz) and at a given storey index  $n$  might be estimated from:

$$L_{v,col}(f, n) = L_{v,base}(f) + \frac{n}{N}C_1(f) + nC_2(f) \quad (\text{dB}) \quad (1)$$

where:

- $L_{v,base}$  is the frequency dependent vibration level at basement level;
- $n$  is the storey index of the floor of interest, from basement ( $n = 0$ ) to roof level ( $n = N$ );
- $N$  is the total number of stories in a building, including any basement levels;
- $C_1$  is a correction factor for the fundamental vertical building mode;
- $C_2$  is a correction factor to account for damping loss at each storey.

The fundamental vertical building mode correction term,  $C_1$  is first defined:

$$C_1(f) = C_{1,1}e^{-\left(\frac{f-f_0}{C_{1,2}}\right)^2} \quad (\text{dB}) \quad (2)$$

where:

- $f_0$  is the fundamental vertical mode frequency, in Hz. This may be known from measurements, or estimated from prediction models of the building;
- $C_{1,1}$  is an amplitude factor, which will represent the total amplification in decibels at the roof level (relative to basement), at  $f_0$ . For the generic concrete frame building model, a value of 9 dB was found to be appropriate.
- $C_{1,2}$  is a frequency factor in Hz, controlling the width of the peak in the correction term at  $f_0$ . For the generic concrete frame building model, a value of 6 Hz was found to be appropriate.

The correction term  $C_2$  accounting for damping loss at each storey can be defined as:

$$C_2(f) = -\log\left(1 + \left(\frac{f}{c_{2,1}}\right) + \left(\frac{f}{c_{2,1}}\right)^4\right) \quad (\text{dB}) \quad (3)$$

where:

$C_{2,1}$  is a frequency factor in Hz, acting like a cut-off frequency for a low-pass filter. For the generic concrete frame building model, a value of 50 Hz is suitable.

The mid-span floor vibration is then calculated using an additional correction term  $C_3$ , accounting for the modal effects of the floors. Thus, it is proposed that the vibration level at mid-span locations,  $L_{v,mid}$  in a given frequency band  $f$  (in Hz) might then be estimated from:

$$\begin{aligned} L_{v,mid}(f, n, f_{1,2,3}) &= L_{v,col}(f, n) + C_3(f, f_{1,2,3}) \\ &= L_{v,base}(f) + \frac{n}{N}C_1(f) + nC_2(f) + C_3(f, f_{1,2,3}) \end{aligned} \quad (\text{dB}) \quad (4)$$

The correction term  $C_3$ , is defined as:

$$C_3(f, f_{1,2,3}) = 3.3C_{3,1} \log\left(1 + e^{-\left(\frac{f-f_1}{c_{3,2}}\right)^2} + 0.5e^{-\left(\frac{f-f_2}{c_{3,2}}\right)^2} + 0.5e^{-\left(\frac{f-f_3}{c_{3,2}}\right)^2}\right) \quad (\text{dB}) \quad (5)$$

where:

$f_{1,2,3}$  are the first three modal frequencies of the floor (in Hz) that have a maximum at the centre of the floor span. These frequencies may be known from measurements, or estimated from prediction models e.g. [9–11]. Unpublished work by this author has shown that for floors with symmetric boundary conditions and with an aspect ratio of 3:4,  $f_2 \sim 4f_1$  and  $f_3 \sim 9f_1$ .

$C_{3,1}$  is an amplitude factor, which will represent the total amplification in decibels at  $f_1$ . A value of 8 dB may be appropriate in the absence of additional guidance from measurements or model data.

$C_{3,2}$  is a frequency factor in Hz, controlling the width of the peaks in the correction term at the modal frequencies. For the generic concrete frame building, a value of 7 Hz is suitable.

## 4. Results

Results from the proposed formulae may be compared against results from the full FE model of the generic building. Results for the one-third octave band vertical vibration level are shown relative to the average basement vibration level in Fig. 4 for column positions, and Fig. 5 for mid-span positions. Note that these average basement vibration levels have been calculated by taking the arithmetic mean over all mid-span or column evaluation positions at the basement level.

For each storey, the mean and mean  $\pm$  one standard deviation ( $\sigma$ ) values shown are averages over all applicable positions on that particular storey, relative to an average basement vibration level. The line labelled “single” denotes the vibration at a single example point on that particular storey relative to its corresponding single point at basement level. Whilst the mean and mean  $\pm \sigma$  metrics provide a representation of vibration distribution through the building, the “single” data is expected to be more representative of a typical on-site measurement.

It should be noted that for the empirical model results, the modal frequencies for the building and floor plates have been estimated from simplified models, not from the full FE model. For example, for each storey the floor modal frequencies were predicted from a simplified FE model of an individual floor plate, with boundary conditions, thickness and material parameters appropriate for the storey under consideration. This approach is considered representative of how the proposed approach might be used in practice. However, the amplitude of some of the empirical constants has been determined from evaluation of full FE building models as well as measurement data.

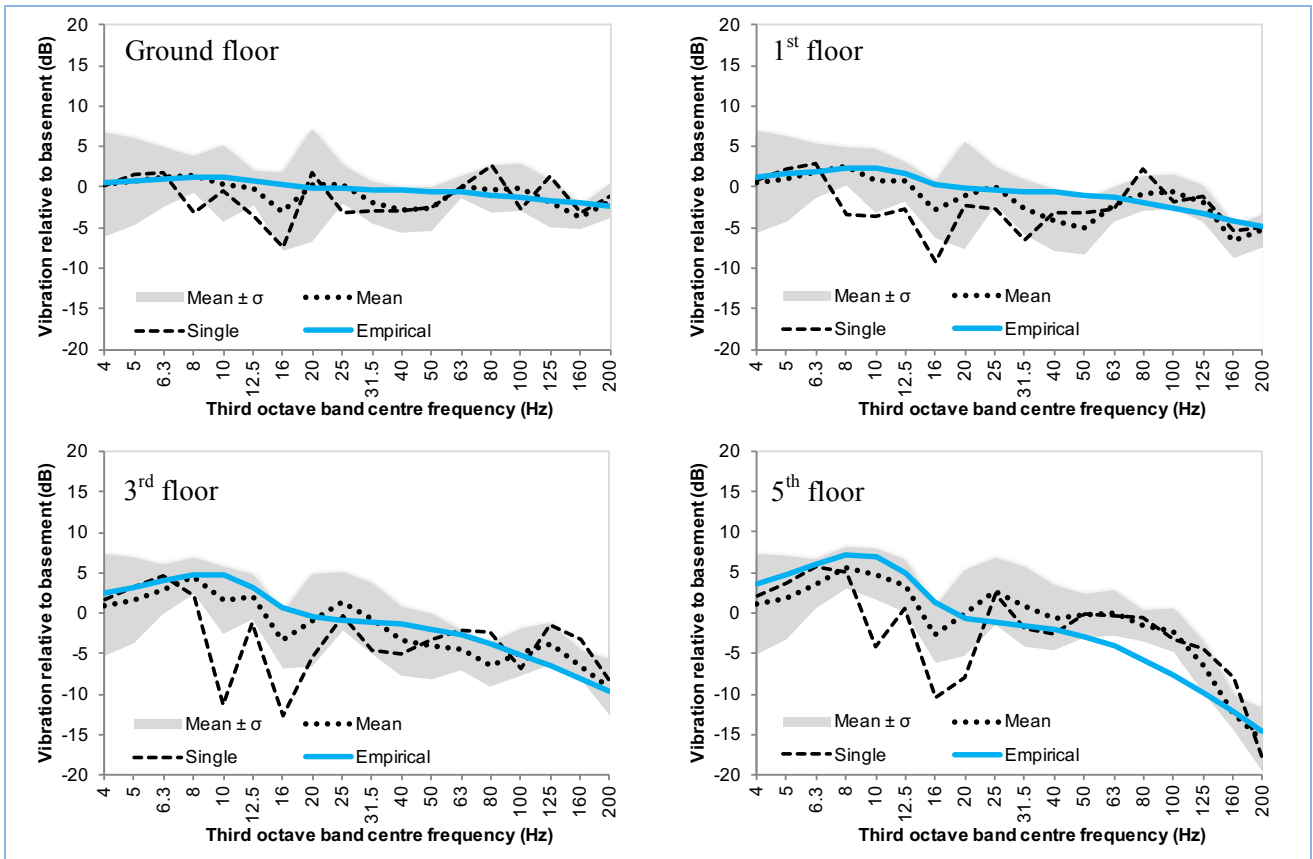


Figure 4. Generic building, FE and proposed empirical predictions, 1/3 octave vertical vibration relative to basement, column positions

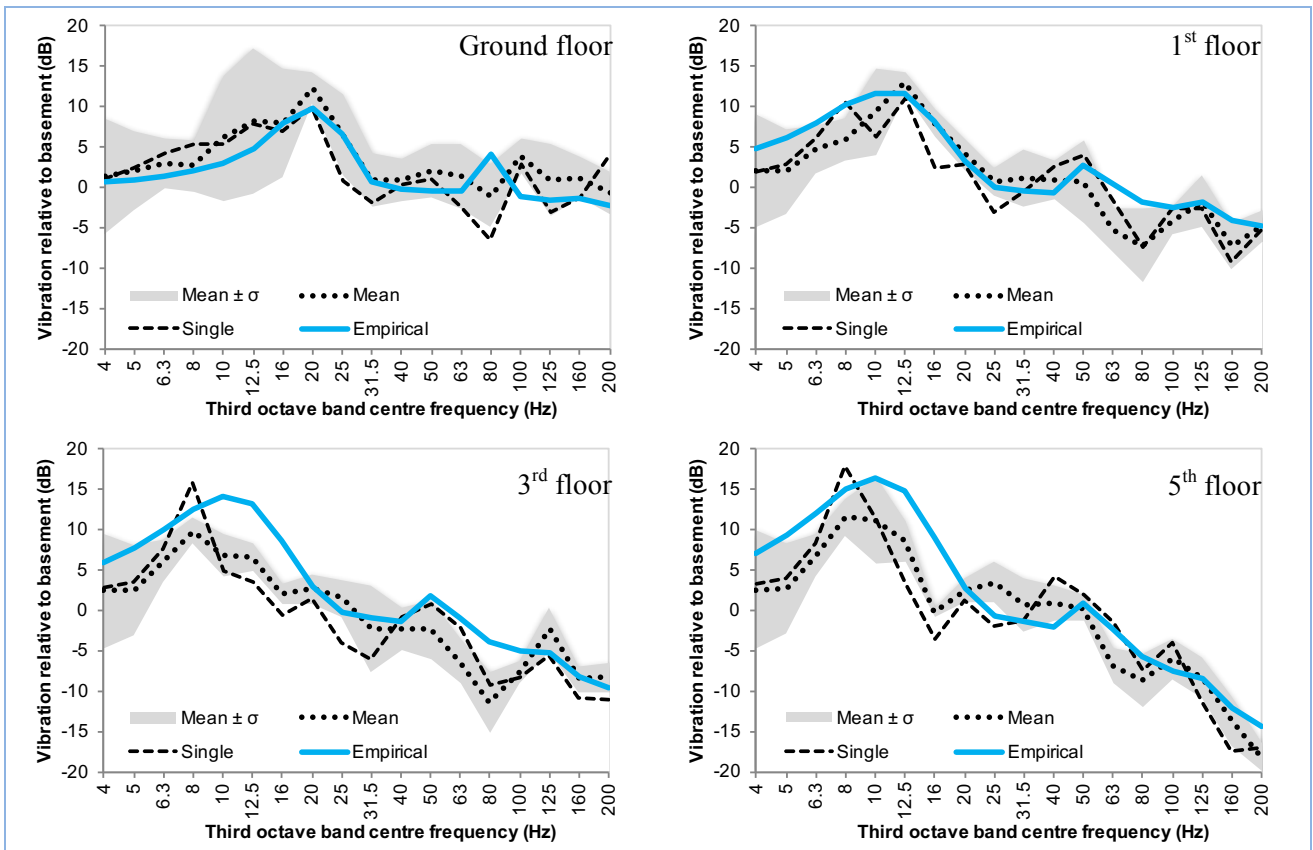


Figure 5. Generic building, FE and proposed empirical predictions, vertical vibration relative to basement, mid-span positions

Fig. 4 shows that, for the column positions, the proposed formulae give results that have a similar shape to the full FE model results. The exception is at the uppermost storey (5<sup>th</sup> floor), at which the empirical model gives results that are a few dB lower between 63 and 125 Hz. This is due to roof effects, which are not included in the empirical model.

Mid-span results in Fig. 5 also show a similar shape when comparing FE and proposed empirical predictions. There are some differences in the floor plate modal frequencies, which suggest that the simplified FE model of a single floor plate may tend slightly to overestimate the resulting modal frequencies, which is likely to be due to approximations made regarding the boundary conditions.

Overall  $W_b$ -weighted acceleration and A-weighted velocity levels, in each case relative to the basement, are plotted in Fig. 6 for column positions, and Fig. 7 for mid-span positions.  $W_b$ -weighted acceleration is used for assessing feelable vibration whereas A-weighted velocity is used for assessing resulting sound pressure levels. The spectra used to determine these overall values are based on the average measured basement spectrum from several buildings affected by groundborne vibration from underground railways, as detailed in [6]. In these figures, additional results are shown for reference:

- “Meas. average” is the average trend observed from measurements of several buildings (see [6]);
- “Meas. similar” is from measurements undertaken on a building which was similar in construction and size to the FE model;
- “TNRB” is based on empirical predictions suggested in the Transportation Noise Reference Book [2].

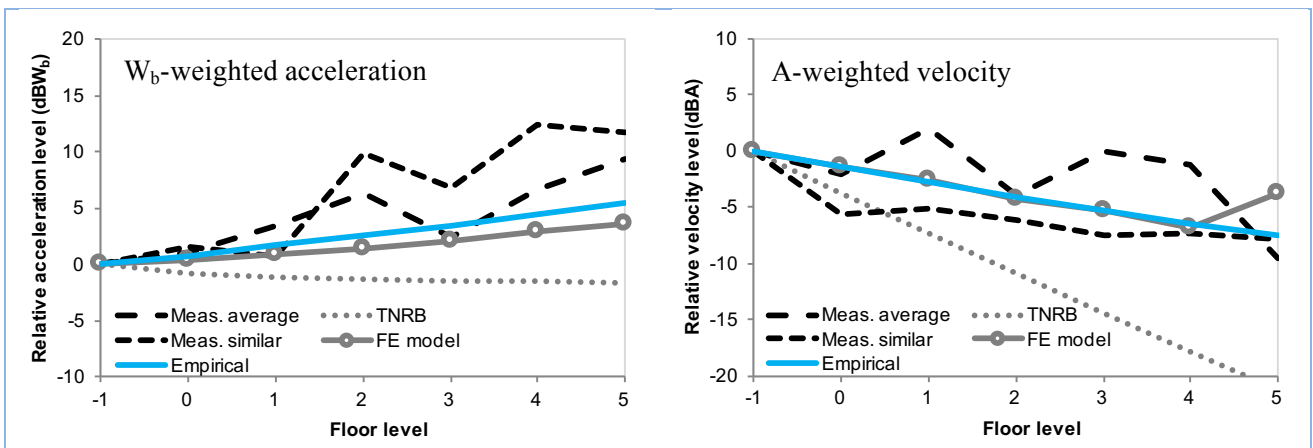


Figure 6. Generic building, FE and proposed empirical predictions, vertical vibration relative to basement, column positions

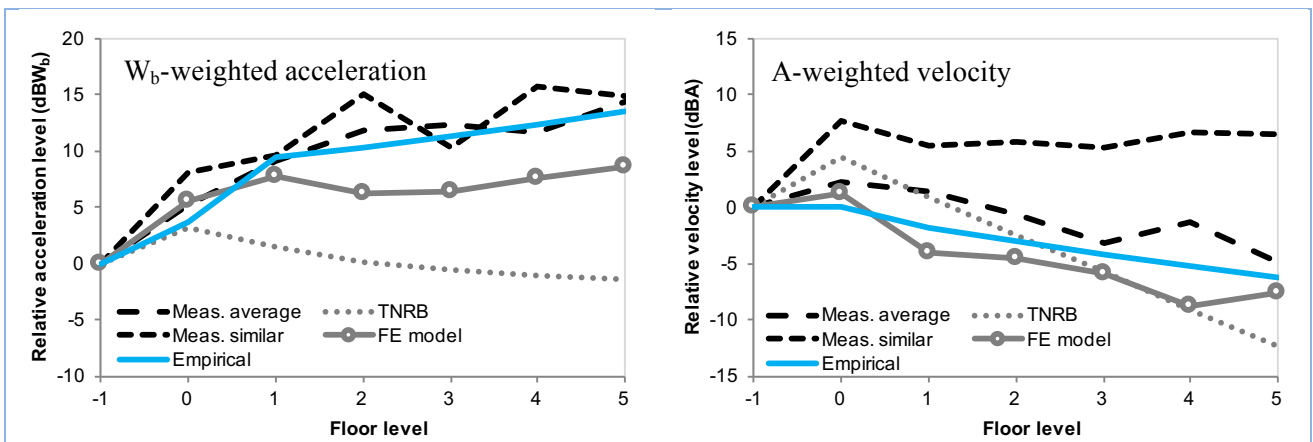


Figure 7. Generic building, FE and proposed empirical predictions, vertical vibration relative to basement, mid-span positions

When considering the column positions, the  $W_b$ -weighted results from the empirical approach are similar to the full FE model results and are typically 2-3 dB below the average measurement trend for locations above 1<sup>st</sup> floor. At 2<sup>nd</sup> floor and above, the measurements for the similar building are somewhat higher than the various predictions, although it is not clear whether this could be due to local vibration effects present in those particular measurements, which included only a single column and a single mid-span position at each storey. The TNRB predictions show a slight decrease in overall vibration with floor level, which differs from all other measured and prediction trends.

For the overall A-weighted column results, the empirical model follows the FE model result closely, except for the uppermost storey, for which roof reflection effects are important but were not included in the empirical model. Both the FE and empirical predictions lie between the measured results. The TNRB approach predicts a greater attenuation per storey than the other predictions or measurements suggest.

The empirical approach predicts mid-span  $W_b$ -weighted vibration that is similar to the measured datasets, whilst the FE model predictions are 3-4 dB less above 2<sup>nd</sup> floor. The FE model results are lower because the influence of individual floor modes is reduced when the response from multiple mid-span positions are averaged over a whole storey. The TNRB approach gives vibration levels that are significantly lower than the other measured or predicted values, and as such this approach should be used with caution when predicting feelable vibration in buildings.

Mid-span A-weighted values are shown to exhibit similar trends in each case. The exception is in the measurements for the similar building, although this is reliant on data measured at a single point in the basement which could include local effects that may bias the results. The proposed empirical results lie between the FE model and average building trend results. The TNRB predictions are also of a similar order, although the attenuation per storey is greater than is observed in the other values; above the 5<sup>th</sup> floor the TNRB predictions are expected to diverge significantly from the other predictions and measurements (towards lower values).

Dynamic soil-structure interaction is not provided for explicitly in the proposed empirical formulae. However, it is included indirectly through allowances for the first building mode frequency etc. in the  $C_1$  term. Whilst recent work by Jin [12] suggests that inclusion of soil-structure interaction could have an influence on the relative vibration levels within a building, the comparisons made are also likely to be due to differences in the force inputs, i.e. a uniformly distributed vertical force over the basement slab vs a wave field in the soil generated by a train in a tunnel. This is especially true at the upper frequencies, which could potentially be under-represented when averaged over the basement level, thereby leading to the appearance of reduced attenuation up the building compared to the uniformly distributed force input case.

It should be noted that the empirical formulae suggested in this paper follow the approximate shape of the most significant amplification factors; regions of the frequency spectrum which exhibit limited amplification are not so well represented, and are typically over-predicted. Nevertheless, by ensuring the most significant amplification effects are included appropriately, the overall weighted values are likely to be similar to results from more detailed models and/or measurements.

## 5. Conclusions

As part of research into the prediction of groundborne noise and vibration in buildings, a parametric study has been conducted on 3D FE models of buildings. This, combined with analysis of measurement data obtained from several buildings affected by underground railways, has been used to inform a new empirical approach for predicting vibration levels within a building. The new prediction approach includes terms to account for building and floor modes, which can be determined from simplified models or measurements.

Predictions employing the new approach show good agreement with an FE building model and with measurements, in most cases lying somewhere between the two. The new approach provides benefits over traditional simple empirical guidance such as that given in the Transportation Noise

Reference Book [2], with the main improvements being increased accuracy and the ability to study the effects of varying structural parameters. These benefits come at a cost of requiring additional empirical terms that may not be readily available.

It is recommended that future work should include providing examples of appropriate terms for various structure types. In addition, this approach may be extended to include terms for acoustic radiation and room acoustics in order to calculate resulting sound pressure levels within buildings.

## ACKNOWLEDGEMENTS

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